

Semi-PROPELLER Compressed Sensing Image Reconstruction with Enhanced Resolution in MRI

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Abstract—Magnetic Resonance Imaging (MRI) reconstruction algorithm using semi-PROPELLER compressed sensing is presented in this paper. It is exhibited that introduced algorithm for estimating data shifts is feasible when super-resolution is applied. The offered approach utilizes compressively sensed MRI PROPELLER sequences and improves MR images spatial resolution in circumstances when highly undersampled k-space trajectories are applied. Compressed Sensing (CS) aims at signal and images reconstructing from significantly fewer measurements than were conventionally assumed necessary. Compressed sensing (CS) aims at signal and images reconstructing from significantly fewer measurements than were traditionally thought necessary. It is shown that the presented approach improves MR spatial resolution in cases when Compressed Sensing (CS) sequences are used. The application of CS in medical modalities has the potential for significant scan time reductions, with visible benefits for patients and health care economics. These methods emphasize on maximizing image sparsity on known sparse transform domain and minimizing fidelity. This diagnostic modality struggles with an inherently slow data acquisition process. The use of CS to MRI leads to substantial scan time reductions and visible benefits for patients and economic factors. In this report the objective is to combine Super-Resolution image enhancement algorithm with both PROPELLER sequence and CS framework. All the techniques emphasize on maximizing image sparsity on known sparse transform domain and minimizing fidelity. The motion estimation algorithm being a part of super resolution reconstruction (SRR) estimates shifts for all blades jointly, emphasizing blade-pair correlations that are both strong and more robust to noise.

Keywords—MRI, super-resolution, compressed sensing.

I. INTRODUCTION

IN typical MRI procedures, several different types of images (e.g. T1, FLAIR, DWI) are being captured within 30 minutes or more. This could be perceived as somewhat inappropriate and uncomfortable. Moreover, due to that inherent patient motion often results in image artifacts. In this background, one can approximate many structures as rigid bodies, with three degrees of freedom applied to the entire MRI volume in a time-varying manner. PROPELLER has been revealed to be quite effective in mitigating patient motion [6], but it is obviously not error-free. Imprecise estimates of motion can lead to corruption of otherwise motion-free data sets, as well as result in non-optimal correction of motion-corrupted data sets. The applied in SRR new shift estimation algorithm [6] is shown to be useful in cases when patient motion is present. Additionally acquiring incomplete k-space data may accelerate the process of collecting of MR measurements. The method recognized as

Compressed sensing exploits the image sparsity information. During the recent years compressed sensing (CS) has gained importance, mostly inspired by the positive theoretical and experimental results shown in [1], [2], [3]. The sparsity in Magnetic Resonance Imaging (MRI) is applied to significantly undersample k-space. Compressed sensing MRI may become an essential medical imaging tool with an inherently slow data acquisition process. Combining CS and MRI offers potentially significant scan time reductions, with benefits for patients and health care economic factors. Technically, MRI requests two major features for successful application of CS:

a) medical imagery is physically compressible by sparse coding in an appropriate transform domain (e.g., by wavelet transform)

b) MRI scanners are able to acquire encoded samples. The compressed sensing theory is available even from the samples collected at lower than the Nyquist rate as long as the unknown image is sparse or compressible.

It is obvious that each imaging device has its own inherent resolution, which is determined based on physical constraints of the system detectors that are in turn tuned to signal-to-noise and timing considerations. Super Resolution (SR) technology has been proved to be useful in medical imaging modalities including Magnetic Resonance Imaging (MRI), functional Magnetic Resonance Imaging (fMRI), Computed Tomography (CT) and Positron Emission Tomography (PET) [5]. Magnetic resonance imaging (MRI) is well known as a non-invasive method routinely used to produce high-quality images of the body's internal tissues. Recently, sampling of MR signals on a Cartesian grid in k-space has been the most common acquisition trajectory. These regularities lead to direct use of the Fast Fourier Transform. It turned out to be too simple sampling model. Currently, non-uniform sampling patterns of the k-space, such as radial, spiral, PROPELLER or SPARSE- SENSE, are gaining their importance in various MRI applications. The MR image reconstruction problem is closely related to the problem of reconstructing a band-limited signal from nonuniformly sampled set of observations in the frequency domain space. The motion of a subject during the MRI acquisition may generate artifacts and blurring in the resulting image. This obstacle may be overcome by using many different k-space trajectories (i.e. PROPELLER or set of CS k- spaces). In case of MRI if the imaging volume is acquired two or more times with small spatial shifts between acquisitions, a combination of data sets by an iterative SR algorithm gives improved resolution and better edge definition in the slice- select direction than simple low-resolution (LR) images averaging as well as native PROPELLER k-

space based reconstruction. The motion correction is still the significant difficulty both to MRI and SR concepts. In this paper as motion analysis core of SR technique a three degrees of freedom-based approach is adopted. Preliminary tests confirmed its usability in PROPELLER MRI [6]. This work goal is to reconstruct a High Resolution (HR) Magnetic Resonance image, from a Compressed Sensing (CS) k-space while keeping all the PROPELLER advantages. The presented framework uses “semi-PROPELLER” sequence because the CS idea is nested in its core. It is shown that, indeed, the new technique enhances the MRI images. In this paper, the authors propose a different image reconstruction method utilizing the CS sampling scheme resulting in the improvement of spatial resolution of MR images. Conducted simulation and experimental studies have revealed that the spatial resolution is clearly enhanced compared to the images achieved in standard CS based on Fourier transform imaging.

II. COMPRESSED SENSING IN MRI

Recently, the application of CS in MRI has been gaining importance in research interest. Compressed Sensing model was first described in the literature of Information Theory and Approximation. The essence of this technique is it measures a small number of random linear combinations of the signal values – much smaller than the number of signal samples nominally representing it. What is crucial, the signal may be reconstructed with sufficient accuracy from these simplified measurements by a nonlinear procedure. In MRI this model refers to a special case of CS, where the sampled linear combinations are directly Fourier coefficients (collecting k-space samples). In these circumstances, CS is claimed to be able to reconstruct MRI image from a small subset of k-space rather than an entire k-space grid. The CS approach requires that the desired image have a sparse representation in a known transform domain and the aliasing artifacts caused by the k-space undersampling are incoherent in that transform domain. Moreover a nonlinear reconstruction must be done to impose both sparsity of the image representation and consistency with the acquired data. CS appeared in the literature of information theory and approximation theory as an abstract mathematical idea [7], [9]. One measures a relatively small number of “random” linear combinations of the signal values (much smaller than the number of signal samples nominally defining it). However, because the underlying signal is compressible, the nominal number of signal samples is a gross overestimate of the “effective” number of “degrees of freedom” of the signal. As a result, the signal can be reconstructed with good accuracy from relatively few measurements by a nonlinear procedure. In MRI, we consider a special case of CS where the sampled linear combinations are simply individual Fourier coefficients (k-space samples). In that setting, CS is claimed to be able to conduct accurate reconstructions from a small subset of k-space, rather than an entire k-space grid. A successful application of CS establishes following requirements:

A. Transform sparsity

The desired image should have a sparse representation in a known transform domain (i.e., it must be compressible by

transform coding.

B. Incoherence of undersampling artifacts

The artifacts in linear reconstruction caused by k-space undersampling should be incoherent in the sparsifying transform domain.

C. Nonlinear reconstruction

The image should be reconstructed by a nonlinear method that requires both sparsity of the image representation and consistency of the reconstruction with the acquired samples.

The first condition is clearly met for MR images, as explained above. The fact that incoherence is important, that MR acquisition can be designed to achieve incoherent undersampling, and the fact that there are efficient and practical algorithms for reconstruction will not, at this point in the article, be at all obvious. Planning a CS scheme for MRI can now be expressed as selecting a subset of the frequency domain that can be efficiently sampled and is incoherent with respect to the sparsifying transform. Before the notion of incoherence will be introduced we should note that narrow optimization of incoherence must not be pushed too far [7]. Some of the most impressive and powerful results about CS assume that one samples a completely random subset of k-space, which indeed gives very low coherence [7]. Though random sampling is an inspiring and instructive idea, sampling a truly random subset of k-space is generally impractical. All the practical sampling trajectories must satisfy hardware and physiological constraints. Hence sampling trajectories must follow smooth lines and curves. Furthermore, a uniform random distribution of samples in spatial frequency does not take into account the energy distribution of MR images in k-space, which is far from uniform. Most energy in MRI is concentrated close to the center of k-space and rapidly decays towards outside of k-space. Therefore, eligible patterns for CS in MRI should have variable density sampling with denser sampling near the center of k-space, matching the energy distribution in k-space. Brain scans are the most common clinical application of MRI. Most brain scans use 2-D Cartesian multislice acquisitions. It has been proven that the brain images exhibit transform sparsity in the wavelet domain [7]. The concepts of CS promise to reduce collection time while improving the resolution of current imagery. A formal approach for reconstruction could be briefly described the in following way. Represent the reconstructed image by a complex vector m , let ψ denote the linear operator that transforms from pixel representation into the chosen representation. Let F_s denote the undersampled Fourier transform, corresponding to one of the k-space undersampling schemes. The reconstructions are obtained by solving the following constrained optimization problem:

Minimize $\|\psi m\|_1$, so that $\|F_s m - y\|_2 < \varepsilon$ where y is the measured k-space data from the MRI scanner and controls the fidelity of the reconstruction to the measured data.

The threshold parameter ε is roughly the expected noise level. The l_1 norm means $\|x\|_1 = \sum_i |x_i|$. Minimizing the l_1 norm of $\|\psi m\|_1$ promotes sparsity [6]. The

constraint $\|F_s m - y\|_2 < \varepsilon$ enforces data consistency. Formally, among all solutions that are consistent with the acquired data, we want to find a solution that is compressible by the transform ψ . It is worth mentioning that when finite differencing is used as the sparsifying transform, the objective becomes the well-known total variation (TV) penalty [5]. The authors in [6] tested the application of CS to brain imaging by acquiring a full Nyquist-sampled data set, which has been retrospectively undersampled. For each slice they selected a different random subset of 80 trajectories from 192 possible trajectories, a speedup factor 2.4. It has been shown that undersampling each slice differently reduces coherence compared to sampling the same way in all slices [6]. CS exhibited both meaningful resolution enhancement over LR at the same scan time, and significant reduction of the aliasing artifacts compared to the linear reconstruction with the same undersampling.

In [9] has been likewise presented, a two-step bootstrap reconstruction approach to derive the temporal principal coefficients. It is worth to be mentioned that an initial k-t SPARSE-SENSE reconstruction with temporal FFT as the sparsifying transform has been implemented.

The l_1 norm minimization problem could be solved using a nonlinear conjugate gradient with backtracking line search. CS exhibited both meaningful resolution enhancement over LR at the same scan time, and significant reduction of the aliasing artifacts compared to the linear reconstruction with the same undersampling.

III. TOEPLITZ STRUCTURED MATRICES AS MR-CS SENSING

Latest advances in compressed sensing theory reveals that sensing matrices whose elements are drawn independently from certain probability distributions guarantee exact recovery of a sparse signal from “an incomplete” number of measurements with high probability.

Due to practical reasons it cannot be formulated in this way. In turn it could take Toeplitz matrix form.

This fact prompted the authors [11] considered Toeplitz block matrices as the sensing matrices. They proved that the probability of perfect reconstruction from a smaller number of filter outputs is also high if the filter coefficients are independently and identically-distributed random variable.

In this paper its applications in semi-PROPELLER MR modality is discussed.

Strictly, the compressive measurement could be expressed by a highly underdetermined system $y = \Phi x$, where the vector $y \in R^n$ what may be interpreted as the samples obtained from the sparse signal $x \in R^N$ by the $M \times N$ sensing matrix Φ .

One of the key obstacles in CS is to formulate “appropriate” matrix Φ . This matrix should allow for exact recovery of x from y with high probability. Candès and Tao [13] provided a sufficient condition for this property.

Precisely, a matrix $\Phi \in R^{n \times N}$ is a “good” CS matrix if it satisfies the restricted isometry property (RIP) of order $3m$, where m is the sparsity of the signal x .

Moreover, Baraniuk et al [12] have shown that matrices whose entries are drawn independently from certain probability distributions satisfy RIP of order $3m$ with probability $\geq 1 - e^{-c_2 n}$ provided that $n \geq c_1 m l$, where $c_1, c_2 > 0$ are some positive constants, and hence are “appropriate” compressed sensing matrices.

Nevertheless, the application of these fully random sensing matrices in medical imaging is still limited due to the structural inconsistency with the encoding matrices present in most medical modalities.

Most imaging systems can be expressed by their point spread functions. Thus it could be prerequisite for employing Toeplitz block encoding matrices.

In this article every single PROPELLER-blade is compressively sensed. The author introduces new term “semi-PROPELLER” to describe the potential method of sensing its sampling sub-structures.

IV. PROBLEM FORMULATION

The major drawback in compressed sensing (CS) is to recovery a vector $x \in R^N$ from its linear measurements y of the form

$$y_i = \langle x, \phi_i \rangle \quad 1 \leq i \leq n, \text{ for } n \ll N.$$

The assumption that x is sparse, and the need for recovery x from a sample y which is much smaller in size than x links to solving a convex program with a suitably chosen sampling basis $\phi_i, 1 \leq i \leq n$.

We can characterize it in given below matrix-vector form:

$$y = \Phi x, \text{ where } \Phi \text{ is an } n \times N \text{ matrix.}$$

In [13] Candès and Tao formulated the restricted isometry property as a condition associated with matrices Φ which provides a guarantee on the performance of Φ in compressed sensing. They indicated that if Φ satisfies RIP of order $3m$ and constant $\delta_{3m} \in (0, 1)$:

$$(1 - \delta_{3m}) \|z\|_2^2 \leq \|\Phi_T z\|_2^2 \leq (1 + \delta_{3m}) \|z\|_2^2 \\ \forall z \in R^{|T|}$$

where $T \subset \{1, \dots, N\}$, $|T| \leq |3m|$, the signal x can be exactly recovered from y by solving the convex optimization $x := a$ subject to $\Phi x = y$.

The Toeplitz block matrix could be characterized in the following way:

$$\Phi = \begin{pmatrix} \Phi_k & \Phi_{k-1} & \cdots & \Phi_2 & \Phi_1 \\ \Phi_1 & \Phi_k & \cdots & \Phi_3 & \Phi_2 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \Phi_{l-1} & \Phi_{l-2} & \cdots & \cdots & \Phi_l \end{pmatrix} \in R^{n \times N}$$

where $l < k$ and the blocks $\Phi_i \in R^{d \times e}$ are themselves (truncated circulant) Toeplitz matrices:

$$\Phi_i = \begin{pmatrix} \phi_p^i & \phi_{p-1}^i & \cdots & \phi_2^i & \phi_1^i \\ \phi_1^i & \phi_p^i & \cdots & \phi_3^i & \phi_2^i \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \phi_{q-1}^i & \phi_{q-2}^i & \cdots & \cdots & \phi_q^i \end{pmatrix} \in R^{n \times N}$$

whose elements $\phi_p^i, \phi_{p-1}^i, \dots, \phi_1^i$ are drawn independently from certain probability distributions $q < p$.

V. SUPER-RESOLUTION IMAGE RECONSTRUCTION

The requirements for better resolution in all medical imaging applications still characterize a very significant research challenge. Most imaging applications highly depend on high-resolution imagery. Enhancing image resolution by improving detector array resolution is not always a possible solution to increasing resolution. For instance, while improvements in semiconductor manufacturing have translated into higher-resolution image sensors, decreasing pixel sizes has a tendency to minimize signal-to-noise ratios (SNR) and light sensitivity. In addition, practical cost and physical constraints limit the ability to change detectors for most legacy imaging systems. To overcome this problem, the image processing community is developing a collection of algorithms known as super-resolution for generating high-resolution imagery from systems having lower-resolution imaging detectors. In 2001 and 2002 preliminary attempts were made to adapt SR algorithms to medical imaginary applications. Early research deals with the magnetic resonance imaging. Medical imaging applications differ from typical photographic imaging in several key aspects. For one thing, as distinct from photography, medical imaging modalities usually use highly controlled illumination of the human subject during image acquisition. As with any imaging system, stronger illumination energy provides to higher signal-to-noise ratios. In the case of medical imaging, though, due to health issues an illumination radiation is limited, thus limiting the SNR to well below that of photographic imaging. For another, imaging speed and robustness is more important in medical imaging applications than in photography. Short examination times limit patient discomfort, radiation dose amounts and minimize imaging artifacts associated with patient movement. Third, unlike photography, the goal of medical imaging is to make possible the making a correct diagnosis, to detect abnormalities rather than take pictures. Therefore, image artifacts are much less tolerable in medical images than in photography. Fortunately, medical imaging devices operate under extremely controlled environments. Researchers can utilize prior knowledge about the patient anatomy to get better image quality. Accurate measurement and map of structure in living tissues is basically limited by the imaging system attributes. Super-resolution methods allow for overcome limitation of the acquisition devices without any modifications within hardware.

VI. DISPLACEMENT ESTIMATION

In SR image reconstruction, the LR images represent different views of the same scene: they are subsampled, mutually (sub-pixel) shifted so that they contain complementary information, hence, they can be merged into a single image with higher resolution. The shift estimation (registration) methods can be divided into two groups. Methods belong to the first operate in the spatial domain space. Algorithms from the second solve the registration issue in the frequency domain space.

Assuming that the relative displacement of LR images has been calculated, samples of “continuous” image (projected by camera lens on its sensor) in nonuniformly spaced sampling

points are obtained. After LR samples mapping onto the image plane computation of samples on the dense HR grid is done. It could be done directly, although more complex iterative procedures result in smaller interpolation errors. The motion estimation is done by the method presented in [6]. In this method it is needed to solve a set of linear equations for the relative shift between each pair of blades. First, $\rho_{ij}(\delta_i, \delta_j)$ is being calculated for each blade pair (i, j) . For N blades, there are $N(N-1)/2$ unique blade pairs. A quadratic approximation of over the region near the maximum yields:

$$\hat{\rho}_{ij}(\delta_i, \delta_j) = a_{ij}(\delta_i - \delta_j)^2 + b_{ij}(\delta_i - \delta_j) + c_{ij}$$

In this method, the parameters for this second-order fit were obtained using a least-squares fit to the 11 points about the maximum value. This algorithm has been designed to jointly maximize the correlation for each blade pair. Once a_{ij} and b_{ij} are known, finding the maximum of the equation given above is achieved by solving

$$\frac{\partial \hat{\rho}_{ij}}{\partial \delta_i} = 2a_{ij}(\delta_i - \delta_j) + b_{ij} = 0$$

Taking the derivative with respect to δ_j functionally yields the same equation. The authors in [6] suggested a few practical modifications before it is solved.

First, equations are weighted based on the maximum value of the blade pair correlation. Regular blade correlation, which is caused by blades corrupted from motion, may result in biased solutions. Its degree and type is highly unpredictable and not well modeled. It has been recommended to multiply each blade pair by:

$$w = e^{-0.5 \left[1 - \frac{\text{Max}(\delta_{ij})}{\sigma} \right]^2}$$

where σ is used to denote a “substantial” change in correlation. The calculated value weights the contribution of the corresponding equation to the final solution by the degree of maximum correlation. A Gaussian function was chosen so that the weights were comparable for blade pairs with “good” correlations near 1.0, and then rapidly approached 0 as the blade pair correlation decreased.

The second applied modification is required because the matrix is not full rank. Because every equation solves for relative displacement, adding a constant value to all elements of the solution vector of σ produces an equally valid solution. Adding the equation:

$$\sum_i \delta_i$$

to the system forces the solution to be unique, with zero average displacement. These two modifications result in a final matrix equation.

It is trivial to extend this to N blades with $N(N-1)/2$ blade pairs. The unknown variables δ_i are then estimated by multiplying the right-hand vector by the Moore-Penrose pseudoinverse of the left-hand matrix. Hence, the final solution for the set of δ_i is biased toward those equations with large a_{ij} (“tight” peaks) and a maximum correlation near 1.

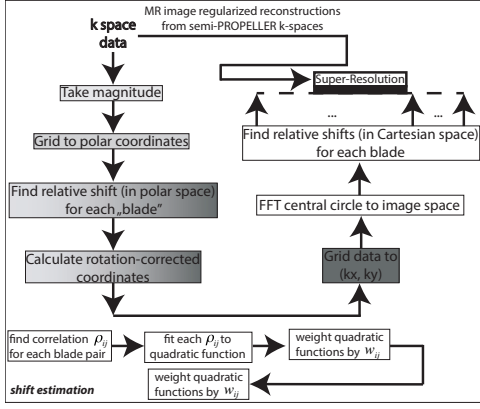


Fig. 1. The algorithm flowchart

VII. THE ALGORITHM

The idea of super-resolution is based on infra-frame motion side information. Precise subpixel image registration is a basic requirement for a good reconstruction, see figure below.

Assuming the PROPELLER blades based low-resolution MR images are accurately registered, the samples of the different images can be combined to reconstruct a high-resolution image. In this paper the projection onto convex sets (POCS) approach [5][10] supported by biharmonic spline interpolation with Green functions at its core has been utilized. The POCS algorithm can be expressed as follows (Malczewski et al., 2006) [5]:

$$g^{k+1} = BRg^k = B[g^k + S_\nu(g - g^k)]$$

where g^k is the reconstructed image after k iterations and S_ν is a sampling operator that extracts values (luminance) on the irregular grid ν . The R (sample replacement operator) and B (ideal low pass filtering) are projections s you really mean something that alternates).

In practice, if a suitable discretization must be applied, last equation is implemented as follows:

$$g_\lambda^{k+1} = B[g_\lambda^k + \chi \vartheta_\chi(g_\lambda - \vartheta_\chi(g_\lambda^k))]$$

Low pass filtering B is realized over a grid of sampling points λ and χ is a convergence and stability parameter. The ϑ_χ is an interpolation operator, here it denotes biharmonic spline scheme. The derivation of the technique in two or more dimensions is similar to the derivation in one dimension. In two dimensions the Green function is equal to:

$$\varphi_2(x) = |x|^2(l)$$

It is worth to be noticed that all the blades are being processed separately. This set of aligned image frames is finally combined into one High Resolution MR image.

VIII. THE EXPERIMENT

All experiments were conducted on a 1.5T MR Signa Excite scanner sequences. All the CS SR reconstruction parts have been implemented in Matlab. Furthermore, CS-semi- PROPELLER and two different linear schemes were applied for

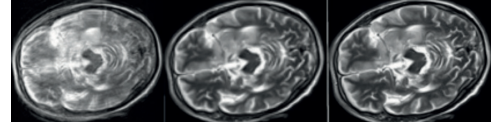


Fig. 2. The Results. From left to right: PROPELLER with no motion correction, PROPELLER with the motion correction [6], CS SR MR image (proposed algorithm).

comparison. The PROPELLER based sequences consisted 23 blades for all slices.

Technically semi-PROPELLER k-spaces have been acquiring by compressive-sensing native PROPELLER blades. The low-resolution acquisition has been included in centric-ordered data with the same number of data samples as the undersampled sets, see figure 2.

Technically, the goal of the simulation was to examine the performance of the CS SR reconstruction and its associated motion artifacts with increased undersampling compared to the LR and Zero Filling with density compensation methods. The further objective was to present the advantage of variable density random undersampling over PROPELLER blades in Super-Resolution. From the “full” irregularly sampled k-space trajectoried sets of randomly undersampled data with uniform density as well as variable density have been constructed. In this trial test a T2-weighted multislice k-space data of a brain has been acquired.

The CS and CS SR MR images have been reconstructed by using proposed algorithms. To demonstrate the image as well as enhanced resolution, the reconstruction algorithms have been compared. Figure 2 shows the simulation results. The LR reconstruction, as expected, shows a poor image quality with visible acceleration of the reconstruction time. In turn, the regular CS exhibits a decrease in apparent SNR because of the incoherent interference. The uniform density undersampling interference is much more visible and more “structured” than the variable density. It is worth to be noticed the CS leads to increased acceleration in comparison to the regular k-space sampling pattern.

IX. THE GOAL OF THE FUTURE RESEARCH

The new SPARSE-SENSING MRI Super-Resolution algorithm has been presented. This report presents the successful use of a super-resolution algorithm to enhance the resolution of MR images. With an increase in scan time for one FOV, a patient trial showed that the super-resolution technique in the axial direction is feasible in a clinical setting without increasing the radiation dose and with no changes in hardware. As expected, the proposed method improves the spatial resolution, but also enhances noise and artifacts. This effect gets more distinctive as the number of super resolution image reconstruction algorithm iterations increases. Preliminary trial results show that the super-resolution approach can be applied to MR imaging, noticeably improving the spatial resolution achievable. The presented algorithm is really simple in use and need no hardware modifications. In general, when applying SR to MRI we can overcome inherent resolution limitations of existing MR imaging hardware. It has been demonstrated that

the sparsity of MR images can be exploited to significantly reduce scan time as well improve the resolution of MR imagery. Conducted simulation and experimental studies have revealed that the spatial resolution is undoubtedly enhanced compared to the images obtained from different sampling schemes. However the motion estimation differs from the original PROPELLER reconstruction it still may suffer from local motion caused image deformations.

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